the study and toil of ages. In all this he holds the noblest of all possessions, a brave, intelligent, and trusting wife, whose sympathy and encouragement is a constant incentive to him to work on, to penetrate still deeper in the hidden mysteries of his laborious science. He is tall, being five feet eleven inches in height, and weighs 185 pounds. He has light grayish blue eyes and a face which leaves the impression of power and capacity.

[Erratum. Page 185, beginning of 16th line from the top of page, insert "to accept the presidency".]

SOME NOVEL AND INTERESTING FORMULAS.

By J. W. NICHOLSON, A. M., LL. D., Member of the London, and New York Mathematical Societies, and President and Professor of Mathematics, Louisiana State University, Baton Rouge, Louisiana.

These formulas are given without demonstration, thinking that their deduction would occupy more space than they probably deserve.

$$(2a+2b)^2+a^2+b^2=(2a+b)^2+(a+2b)^2 \ldots\ldots (1).$$

This is a simple formula for finding three square numbers whose sum is equal to the sum of two squares. Thus, for $a=5$, $b=3$, we have

$$16^2+5^2+3^2=13^2+11^2 \ldots\ldots (2).$$

$$(3a+3b)^n+(2a+4b)^n+a^n+b^n=(3a+4b)^n+(a+3b)^n+(2a+b)^n \ldots\ldots (3),$$

where $n=3$, 2 or 1.

Thus, for $a=5$, $b=3$, we have $24^n+22^n+5^n+3^n=27^n+14^n+13^n \ldots (4)$, where $n=3$, 2 or 1.

$$(5a+10b)^n+(4a+11b)^n+(3a+5b)^n+(2a+8b)^n+(3a+3b)^n+(2a+6b)^n$$
$$+a^n+b^n=(5a+11b)^n+(4a+6b)^n+(3a+10b)^n+(3a+8b)^n+(a+5b)^n+(2a+3b)^n$$
$$+(2a+b)^n \ldots (5),$$ where $n=5$, 4, 3, 2 or 1.

Thus for $a=5$, $b=2$, we have

$$45^n+42^n+26^n+25^n+22^n+21^n+5^n+2^n=47^n+35^n+32^n+31^n+16^n+15^n$$
$$+12^n \ldots (6),$$ where $n=5$, 4, 3, 2 or 1.

In (5) for $a=8$, $b=3$, we find $15^n+10^n+9^n+6^n=14^n+13^n+7^n+3^n+2^n+1^n \ldots (7)$, where $n=5$, 3 or 1.

$$(a+32)^n+(a+24)^n+(a+18)^n+(a+10)^n+(a+4)^n+(a-4)^n+(a-10)^n$$
$$+(a-18)^n+(a-24)^n+(a-32)^n=(a+30)^n+(a+28)^n+(a+16)^n+(a+8)+^n(a+6)^n$$
$$+(a-6)^n+(a-8)^n+(a-16)^n+(a-28)^n+(a-30)^n \ldots (8),$$

where $n=5$, 4, 3, 2 or 1.

In (8) by making $a=7$, we find

$$39^n+31^n+21^n+9^n=37^n+35^n+15^n+13^n \ldots (9),$$ where $n=5$, 3 or 1.

$$j=\underline{|n}=n^n-n(n-1)^n+\frac{n(n-1)}{2}(n-2)^n-\frac{n(n-1)(n-2)}{2.3}(n-3)^n+\&c. \ldots (10),$$

where n is any positive integer.

Thus, for $n=5$, we have

$1.2.3.4.5=5^5-5(4)^5+10(3)^5-10(2)^5+5(1)^5 \ldots\ldots (11).$

Some new and interesting properties of prime numbers.

If $n+1$ is a prime number, then

$(a+nb)^x+[a+(n-1)b]^x+[a+(n-2)b]^x+\ldots\ldots(a+b)^x+a^x=m(n+1)\ldots\ldots(12),$

where m is an integer and x any integer less than n.

In (12) for $a=0$ and b=1, we have

$$n^x+(n-1)^x+(n-2)^x\ldots\ldots 2^x+1=m(n+1)\ldots\ldots(13).$$

Thus, 11 will exactly divide

$10^x+9^x+8^x+7^x+6^x+5^x+4^x+3^x+2^x+1\ldots\ldots(14)$ where $x=9, 8, 7, 6, 5, 4, 3, 2, 1$.

The converse of formulas (12) or (13) is not always true, but the following are true only when $n+1$ is a prime number.

$$(a+n)^n+(a+n-1)^n+(a+n-2)^n+\ldots\ldots a^n+1=m(n+1)\ldots\ldots(15).$$

Making $a=0$, we have

$$n^n+(n-1)^n+(n-2)^n\ldots\ldots 1^n+1=m(n+1)\ldots\ldots(16).$$

That is, S_n+1 is divisible by $n+1$ when it is a prime number and only when it is prime. So far as I know this furnishes an entirely *new criterion* of *prime numbers.*

NOTE. The preceding formulas are taken from a paper, by the author, on "The nth power of any number expressed as the sum of the nth powers of other numbers, n being any positive integer;" which was read before the New York Mathematical Society, Dec. 3d, 1892.

NON-EUCLIDEAN GEOMETRY: HISTORICAL AND EXPOSITORY.

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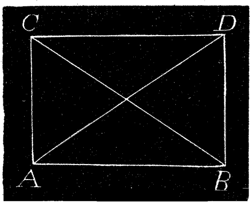
CHAPTER SECOND.

THE FIRST TREATISE ON NON-EUCLIDEAN GEOMETRY.

[Continued from the May Number.]

PROPOSITION I. *If two equal straights* [sects] (*fig* 1.) AC, BD, *make with the straight AB angles equal toward the same parts: I say that the angles at the join CD will be mutually equal.*



PROOF. Join AD, CB. Then consider the triangles CAB, DBA. It follows (Eu. I. 4.) that the bases CB, AD will be equal.

Then consider the triangles ACD, BDC. It